Dynamical Generation of Topologically Massive Gauge Fields and Composite Fermions

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A new dynamical gauge-invariant mass generation mechanism for gauge fields and dynamical generation of composite fermions are found in (2+1)-dimensional gauge non-linear sigma models with fermions within the 1/N expansion.

1. Higgs mechanism [1] is nowadays the most popular mass generation mechanism for gauge fields via spontaneous breakdown of the local gauge symmetry. However, more rigorous treatments [2] have shown recently that due to non-perturbative effects standard Higgs mechanism is inconsistent in the exact quantum theory. More precisely, the vacuum expectation value $\langle F\rangle$ of any gauge-noninvariant quantity F should <u>vanish</u>. Even before Higgs mechanism was proposed, Schwinger [3] observed that massive gauge fields and gauge invariance could perfectly well coexist and a gauge-invariant mass generation mechanism was explicitly realized in the case of D=2 QED (D being the space-time dimension) due to the axial anomaly. Recently the generalization of Schwinger mechanism to D $\geqslant$3 was widely discussed [4,5]. In ref.[5] the novel properties of D=3 QED, QCD (and also, of D=3 Einstein gravity) with <u>explicitly added</u> topological gauge-invariant mass terms (TGIMTs) (abelian and/or non-abelian):

$$\mathcal{L}_{TGIMT} = \frac{\xi_0}{4}\varepsilon_{\mu\nu\lambda} A^{\mu}_{(0)} F^{\nu\lambda}_{(A_{(0)})} + \frac{\xi_1}{4}\varepsilon_{\mu\nu\lambda}\, tr\left(\underset{\sim}{A}^{\mu}\underset{\sim}{F}^{\nu\lambda}_{(\underset{\sim}{A})} - i\,\frac{2}{3}\underset{\sim}{A}^{\mu}\underset{\sim}{A}^{\nu}\underset{\sim}{A}^{\lambda}\right) \qquad (1)$$

were studied in great detail. The name "topological" comes from the close relation of (1) (more precisely, of the second term on the r.h. side of (1)) to the so called topological secondary characteristic classes of Chern-Simons in fibre bundle theory (cf. [5]) and, correspondingly, to the well known instanton density of D=4 Euclidean Yang-Mills theories [6].

Then the natural question arises about the possibility of a <u>dynamical generation</u> of TGIMTs (1) (i.e. due to quantum, eventually non-perturbative, effects). This is the first topic discussed in the present report. Although TGIMTs break P- and T- (space- and time-reflection) invariance it is shown that P- and/or T-<u>noninvariance</u> of the initial classical theory are <u>neither</u> necessary <u>nor</u> sufficient for dynamical generation of TGIMTs.

The second topic concerns non-perturbative particle spectra, i.e. the lacking of a necessary one-to-one correspondence between fields in the (classical) Lagrangian and physical particles ("confinement" of some fundamental fields and/or dynamical generation of bound states).

2. Here we consider D=3 gauge non-linear sigma models with fermions ($[GNLSM\text{-}F]_3$) possessing internal U(N)("flavor")xU(n)("color" gauge) sym-

metry ($n < N$) within the 1/N expansion (GNLSM-F for D=2 were introduced in [7]). The corresponding Lagrangian reads:

$$\mathcal{L}_{\mathrm{GNLSM+F}} = |\nabla_\nu \varphi|^2 + \frac{i}{2}\bar{\psi}\overleftrightarrow{\not\partial}\psi + \frac{\lambda_0}{4Nn\mu}(\bar{\psi}\psi)^2 + \frac{\lambda_1}{4Nn\mu}(\bar{\psi}\tau_A\psi)^2 + Nn\mathcal{L}_A$$
$$\varphi^*\varphi - Nn\mu/T = 0,\ \varphi^*\tau_A\varphi = 0,\ \bar{\psi}\varphi = \varphi^*\psi = \bar{\psi}\tau_A\varphi = \varphi^*\tau_A\psi = 0 \tag{2}$$
$$\mathcal{L}_A \equiv -\frac{1}{4e_0^2\mu}F^2_{\varkappa\lambda}(A_{(0)}) - \frac{1}{4ne_1^2\mu}\mathrm{tr}\left(\underset{\sim}{F}^2_{\varkappa\lambda}(\underset{\sim}{A})\right) - B_{(0)}\partial_\mu A^\mu_{(0)} - \frac{1}{n}\mathrm{tr}\left(\underset{\sim}{B}\partial_\mu \underset{\sim}{A}^\mu\right) + \frac{1}{(Nn)}\mathrm{tr}\left(\underset{\sim}{\chi}^*\partial^\mu\nabla_\mu\underset{\sim}{\chi}\right)$$

with the following notations:

$$F^{\mu\nu}(A_{(0)}) = \partial^\mu A^\nu_{(0)} - \partial^\nu A^\mu_{(0)},\quad \underset{\sim}{F}^{\mu\nu}(\underset{\sim}{A}) = \partial^\mu \underset{\sim}{A}^\nu - \partial^\nu \underset{\sim}{A}^\mu + i[\underset{\sim}{A}^\mu, \underset{\sim}{A}^\nu],$$

$(\nabla_\mu\varphi)^k_a = \partial_\mu\varphi^k_a + iA_{(0)\mu}\varphi^k_a + i\underset{\sim}{A}^{kl}_\mu\varphi^l_a$ (the same for $(\nabla_\mu\psi)^k_a$), $\nabla_\mu\underset{\sim}{\chi} = \partial_\mu\underset{\sim}{\chi} + i[\underset{\sim}{A}_\mu, \underset{\sim}{\chi}]$,

$\not{O} \equiv O^\nu\gamma_\nu$, $\bar{O} \equiv O^*\gamma_0$, $\underset{\sim}{O}^{kl} \equiv O_A\tau_A$, $\mathrm{tr}(\tau_A\tau_B) = n\delta_{AB}$, $\varphi^*\varphi \equiv \varphi^{*k}_a\varphi^k_a$ etc.

In (2) μ is an arbitrary mass scale, so that all coupling constants T, $\lambda_{0,1}$, $e_{0,1}$ are set dimensionless. The auxiliary fields $B_{(0)}$, $\underset{\sim}{B}$ enforce Landau gauge conditions for the U(1) and SU(n) gauge fields $A^\mu_{(0)}$, $\underset{\sim}{A}^\mu$; $\underset{\sim}{\chi}$ are the corresponding Faddeev-Popov ghost fields. The hermitian n x n matrices τ_A, A=1,...,n^2-1, span a hermitian basis of the Lie algebra of SU(n). Summation over repeated indices ("flavor" ones a, b =1,...,N; "color" ones k, l =1,...,n; adjoint-SU(n) ones A,B=1,...,n^2-1 and Lorentz ones) is understood and the latter will be suppressed for brevity. In particular, for $e_{0,1} = \infty$, $\lambda_0 = \lambda_1$ =T $[\mathrm{GNLSM+F}]_3$ coinside with D=3 <u>supersymmetric</u> generalized non-linear sigma models [8]:

$$\mathcal{L}_{\mathrm{SS\ GNLSM}} = |\nabla_\mu\varphi|^2 + \frac{i}{2}\bar{\psi}\overleftrightarrow{\not\partial}\psi + (4Nn\mu)^{-1}T(\bar{\psi}\psi)^2 + (4Nn\mu)^{-1}T(\bar{\psi}\tau_A\psi)^2$$
$$\varphi^*\varphi - Nn\mu/T = 0,\ \varphi^*\tau_A\varphi = 0,\ \bar{\psi}\varphi = \varphi^*\psi = \bar{\psi}\tau_A\varphi = \varphi^*\tau_A\psi = 0 \tag{2'}$$
$$A^\nu_{(0)} = \frac{T}{2Nn\mu}\left[i\varphi^*\overleftrightarrow{\partial^\nu}\varphi + \bar{\psi}\gamma^\nu\psi\right],\quad A^\nu_B = \frac{T}{2Nn\mu}\left[i\varphi^*\tau_B\overleftrightarrow{\partial^\nu}\varphi + \bar{\psi}\tau_B\gamma^\nu\psi\right]$$

To construct the 1/N expansion of the quantum $[\mathrm{GNLSM+F}]_3$ we use the standard trick of converting the Lagrangian (2) into a quadratic function in φ, ψ by means of introducing a set of auxiliary fields ($\alpha_{(0)}$, $\underset{\sim}{\alpha}$, $\sigma_{(0)}$, $\underset{\sim}{\sigma}$ - real bosonic, $\rho_{(0)}$, $\underset{\sim}{\rho}$ - complex fermionic):

$$\mathcal{L}'_{\mathrm{GNLSM+F}} = |\nabla_\mu\varphi|^2 - \varphi^*(\alpha_{(0)} + \underset{\sim}{\alpha})\varphi + \frac{Nn\mu}{T}\alpha_{(0)} - \frac{Nn\mu}{\lambda_0}\sigma^2_{(0)} - \frac{N\mu}{\lambda_1}\mathrm{tr}(\underset{\sim}{\sigma}^2) +$$
$$+ \frac{i}{2}\bar{\psi}\overleftrightarrow{\not\partial}\psi + \bar{\psi}(\sigma_{(0)} + \underset{\sim}{\sigma})\psi + \bar{\psi}(\rho_{(0)} + \underset{\sim}{\rho})\varphi + \varphi^*(\bar{\rho}_{(0)} + \underset{\sim}{\bar{\rho}})\psi + Nn\mathcal{L}_A. \tag{3}$$

Under P- and T-reflection the fields in (2),(3) transform as:

$$\varphi^{(P,T)}_{(x)} = \eta_{P,T}\varphi(x_{P,T}),\ \psi^{(P,T)}_{(x)} = -i\eta_{P,T}\gamma_{1,2}\psi(x_{P,T}),\ \rho^{(P,T)}_{(x)} = i\gamma_{1,2}\rho(x_{P,T})\ \alpha^{(P,T)}_{(x)} = \alpha(x_{P,T})$$

$$\sigma^{(P,T)}_{(x)} = -\sigma(x_{P,T}),\ A^{(P)}_\mu(x) = (A_0 - A_1, A_2)(x_P),\ A^{(T)}_\mu(x) = (A_0, -A_1, -A_2)(x_T),$$

$$(x^\mu_P) \equiv (x^0, -x^1, x^2),\ (x^\mu_T) \equiv (-x^0, x^1, x^2),\ \eta_{P,T} = \pm 1$$

Thus $[\mathrm{GNLSM-F}]_3$ are P- and T-invariant. Note that P,T-reflections in D=3 are analogues of the discrete chiral γ_5-transformation in D=4 (in particular, $\bar{\psi}^{(P,T)}\psi^{(P,T)} = -\bar{\psi}\psi$ (D=3)).

3. Following standard techniques (see e.g. [9]) the fields ψ, $\varphi_\perp$, where $\varphi = N^{1/2}\varphi_\parallel + \varphi_\perp$, $\varphi^K_{\perp a_1} = 0$, $a_1 = 1,\dots,n$, $\varphi^K_{\parallel a_2} = 0$, $a_2 = n-1,\dots,N$, are integrated out in the generating functional for the Green's functions of (3):

$$Z[\text{sources}] = \text{const}\int D\varphi_\perp D\psi D\varphi_\parallel D\alpha_{(0)}\cdots D\chi \exp\Big\{i\int d^3x\Big[\mathcal{L}'_{\text{NGLSM+F}} + \text{source terms}\Big]\Big\} = (\text{const})'\int D\varphi_\parallel D\alpha_{(0)}\cdots D\chi \exp\Big\{iN S_1 + i\,\mathcal{S}_2[\text{sources}]\Big\};\quad(4)$$

$$S_1 \equiv i\left(1-\frac{n}{N}\right)\mathrm{Tr}\ln\Delta_B - i\,\mathrm{Tr}\ln\Delta_F + \int d^3x\Big[-\varphi^*_\parallel\Delta_B\varphi_\parallel + n\mu/T\,\alpha_{(0)} - n\mu/\lambda_0\,\sigma^2_{(0)} - \mu/\lambda_1\,\mathrm{tr}(\underset{\sim}{\sigma}^2) + n\mathcal{L}_A\Big],\quad(5)$$

$$\Delta_F \equiv i\not{\nabla} + \sigma_{(0)} + \underset{\sim}{\sigma},\quad \Delta_B \equiv \nabla_\mu\nabla^\mu + \alpha_{(0)} + \underset{\sim}{\alpha} + (\bar\rho_{(0)} + \underset{\sim}{\bar\rho})\Delta_F^{-1}(\rho_{(0)} + \underset{\sim}{\rho});$$

and the 1/N expansion is generated through expansion of (4) around the constant saddle points of S_1(5):

$\hat\alpha_0 \equiv m^2_\varphi$, $\hat\sigma_{(0)} \equiv -m_\psi$, $\varphi_\parallel \equiv V$ (all other fields having zero stationary values). The stationarity equations for S_1 possess the following distinct types of solutions:

(i) "High-temperature" phase solution ($T > T_c$):

$$V = 0,\ m_\varphi = 4\pi\mu(1/T_c - 1/T),\ m_\psi = 4\pi\mu(1/T_c - 1/\lambda_0)\,\Theta(\lambda_0 - T_c);\quad(6.a)$$

$$|V|^2\delta^{\chi\ell} \equiv V_a^{*K}V_a^\ell,\quad T_c \equiv 4\pi(1+a_0),\quad \Theta(y) = \begin{cases}1 & y>0\\ 0 & y<0\end{cases}$$

where a_0 is an arbitrary constant accounting for the arbitrariness in the renormalization of divergent point-loop graphs (cf.[8]);

(ii) "Low-temperature" phase solution ($T < T_c$):

$$|V|^2 = \mu(1/T - 1/T_c),\ m_\varphi = 0,\ m_\psi = 4\pi\mu(1/T_c - 1/\lambda_0)\,\Theta(\lambda_0 - T_c);\quad(6.b)$$

(iii) "Critical" theory ($T = T_c$):

$$V = 0,\ m_\varphi = 0,\ m_\psi = 4\pi\mu(1/T_c - 1/\lambda_0)\,\Theta(\lambda_0 - T_c)\quad(6.c)$$

Now, from the quadratic part of S_1(5) the "free" propagators ($\langle\dots\rangle^{(0)}$) of the 1/N expansion can be found. We write down the latter in a unified notation (in momentum space) simultaneously suited for (i),(ii),(iii) (in each separate case V, m_φ, m_ψ are to be set equal to their corresponding values (6.a,b,c)). For the fundamental fields in (2) we have:

$$\langle\varphi_a^K\varphi_b^{*\ell}\rangle^{(0)} = (-i)\delta_{ab}\delta^{K\ell}[m^2_\varphi - p^2]^{-1} + \frac{1}{n}(-p^2)^{-2}\Big\{V_a^K V_b^{*\ell} Nn\langle\alpha_{(0)}\alpha_{(0)}\rangle^{(0)} + (\tau_a V_a)^K(V_b^*\tau_B)^\ell Nn\langle\alpha_A\alpha_B\rangle^{(0)}\Big\};\quad(7.a)$$

$$\langle\psi_a^K\bar\psi_b^\ell\rangle^{(0)} = (-i)\delta_{ab}\delta^{K\ell}(\not{p} + m_\psi)[m^2_\psi - p^2]^{-1} + \frac{1}{n}(-p^2)^{-2}\Big\{V_a^K V_b^{*\ell} Nn\langle\rho_{(0)}\bar\rho_{(0)}\rangle^{(0)} + (\tau_A V_a)^K(V_b^*\tau_B)^\ell Nn\langle\alpha_A\alpha_B\rangle^{(0)}\Big\};\quad(7.b)$$

$$\langle A^\mu_{(0)}A^\nu_{(0)}\rangle(\ell_0) = (Nn)^{-1}i\,[\mathcal{F}^2(\ell_0) - p^2G^2]^{-1}\Big\{\Big(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\Big)\mathcal{F}(\ell_0) + i\varepsilon^{\mu\nu\lambda}p_\lambda G\Big\}\quad(7.c)$$

$$\mathcal{F}(e) = -p^2/e^2\mu + |V|^2 + (4m^2_\varphi - p^2)\tfrac{1}{2}F(p^2;m_\varphi) - (4m^2_\psi + p^2)\tfrac{1}{2}F(p^2;m_\psi) + 1/4\pi(m_\psi - m_\varphi),\quad G \equiv 2m_\psi F(p^2;m_\psi) = 1/4\pi\, f(p^2/4m^2_\psi)\quad(7.d)$$

$$F(p^2;m) = (8\pi m)^{-1}f(p^2/4m^2),\quad f(z) = (2\sqrt{z})^{-1}\ln[(1+\sqrt{z})(1-\sqrt{z})^{-1}];\quad(7.e)$$

$$\langle A^\mu_B A^\nu_C\rangle^{(0)}(e_1) = \delta_{BC}\langle A^\mu_{(0)}A^\nu_{(0)}\rangle^{(0)}(e_1)\quad(7.f)$$

The propagators of the auxiliary fields read:

$$\langle \rho_{(0)} \bar{\rho}_{(0)} \rangle^{(0)} = 2i(Nn)^{-1}\left[(\not p + 2m_\psi)F(p^2; \tfrac{1}{2}(m_\varphi + m_\psi))\right]^{-1}, \; \langle \rho_A, \rho_B \rangle^{(0)} = \delta_{AB} \langle \rho_{(0)} \bar{\rho}_{(0)} \rangle^{(0)} \quad (8.a)$$

$$\langle \alpha_{(0)}, \alpha_{(0)} \rangle^{(0)} = i(Nn)^{-1}\left[F(p^2; m_\varphi) - 2|v|^2/p^2\right]^{-1}, \; \langle \alpha_A \alpha_B \rangle^{(0)} = \delta_{AB} \langle \alpha_{(0)} \alpha_{(0)} \rangle^{(0)} \quad (8.b)$$

$$\langle \sigma_{(0)} \sigma_{(0)} \rangle^{(0)} = -i(Nn)^{-1}\left[(4m_\psi^2 - p^2)F(p^2; m_\psi)\right]^{-1} \quad (8.c)$$

$$\langle \sigma_A \sigma_B \rangle^{(0)} = -i(Nn)^{-1}\left[(4m_\psi^2 - p^2)F(p^2; m_\psi) + 2\mu(1/\lambda_1 - 1/\lambda_0)\right]^{-1} \quad (8.d)$$

4. Now, from (6)-(8) the properties of the corresponding phases of $[\mathrm{GNLSM+F}]_3$ (within the 1/N expansion) can be directly derived.

The non-zero dynamical fermion mass $m_\psi(\sigma)$ gives rise to dynamical P,T-breakdown in both phases.

In the "low-temperature" phase spontaneous breaking of the internal $U(N)xU(n)_{gauge}$ symmetry takes place leaving a residual

$$U(N-n) \times \mathrm{diag}(U(n)_{global} \times U(n)_{gauge})$$

symmetry (stability subgroup of the "low-temperature" phase vacuum: $\langle \varphi^K_{\shortparallel a} \rangle = N^{\frac{1}{2}}(V^K_a + O(N^{-1}))$ and the corresponding particle spectrum consists of only n(N-1) massless Goldstone bosons $(\varphi_\perp, V^{*\ell}_a \varphi^K_a, K \neq \ell)$ and of n(N-1) (massive) fermions $(\psi_\perp, V^{*\ell}_a \psi^K_a, K \neq \ell)$ All the remaining fields are here "confined" due to $\sqrt{-p^2}$ - singularities in their propagators and, in particular, standard Higgs mechanism is suppressed.

The "high-temperature" phase is $U(N)xU(n)_{gauge}$ symmetric and its particle spectrum consists of n.N pairs of bosons (φ^K_a) and fermions (ψ^K_a) with dynamically generated masses m_φ, m_ψ (6) and of the following set of dynamically generated (bound) states:

(α) Massive gauge bosons $A^\mu_{(0)}$, A^μ_B due to dynamically generated TGIMTs (1) with $\xi_0 = n\xi_1$ =Nn/4π, which shown up in (7.c.f) as:

$$-(Nn)^{-1} \varepsilon^{\mu\nu\lambda} p_\lambda \frac{1}{4\pi} f(p^2/4m_\psi^2)\left[\mathcal{F}^2(e_0, 1) - p^2 G^2\right],$$

in the following regimes $(\eta \equiv m_\varphi/m_\psi)$:

$$\text{either } \eta > 1, \; e^2_{0,1} < 16\pi m_\psi \left\{\mu(\eta-1)\left[(\eta+1)\eta^{-1} f(\eta^{-2}) - 1\right\}^{-1}\right.$$

$$\text{or } \eta < 1, \; e^2_{0,1} < 16\pi m_\psi \eta^2 \left\{\mu(1-\eta)\left[1 - (1-\eta) f(\eta^2)\right]\right\}^{-1} \quad (9)$$

and the corresponding masses are obtained as solutions of the transcendental equations (cf. (7.c,f)):

$$2\sqrt{x}\, f(x) = (1+x) f(x) + 16\pi m_\psi (\mu e^2_{0,1})^{-1} x - 1 + \eta - \eta(1 - x/\eta^2) f(x/\eta^2),$$
$$x \equiv m^2_{A(0),B}/4m^2_\psi, \quad m_{A(0),B} < \min\{2m_\psi, 2m_\varphi\};$$

(β) Massive composite fermions $\rho_{(0)}$, ρ_A in the regime $m_\varphi > m_\psi$ with $m_{\rho(0),A} = 2m_\psi$ (cf. (8.a), (7.e)). Recall that classically:

$$\rho_{(0)} = -T(Nn\mu)^{-1}(\varphi^* i\not\partial \psi), \; \rho_A = -T(Nn\mu)^{-1}(\varphi^* \tau_A i \not\partial \psi); \quad (10)$$

(γ) Massive fermion condensate bound states $\sigma_A = \frac{\lambda_1}{2N\pi\mu}(\bar{\psi}\tau_A\psi)$ in the regime $m_\psi/4\pi\mu < 1/\tau_c - 1/\lambda_1 < m_\psi/2\pi\mu$ with a mass obtained as solution of the transcendental equation:

$$(1-x)f(x)+1+\frac{4\pi\mu}{m_\psi}\left(1/\lambda_1 - 1/\tau_c\right)=0, \quad 0\leq x \equiv m^2_{\sigma_A}/4m^2_\psi < 1$$

(α), (β), (γ) follow directly by using the explicit form of $f(z)$ (7.e). Note that the (would-be) pole of $\langle\sigma(0)\sigma(0)\rangle^{(0)}$ at $p^2=4m^2_\psi$ does not correspond to a real stable particle since $F(p^2; m_\psi)$ (7.e) has a logarithmic branching point there ($\psi - \psi$ threshold).

Now, some remarks are in order. Higher orders in 1/N will in general shift the values of all masses, however, the structure of the particle spectra remains valid, provided the 1/N expansion of $[\mathrm{GNLSM+F}]_3$ were true as an asymptotic series. To prove the latter, however, is a highly nontrivial task (asymptotic convergence of 1/N expansion was proved so far only for some much more simple models [10]).

The above treatment within the 1/N expansion can be applied also to the general class of D=3 Higgs models with fermions (HMF_3) which break P- and T-invariance already on classical level (due to terms like $(\bar{\psi}\psi)(\varphi^*\varphi)$, $(\bar{\psi}\varphi)(\varphi^*\psi)$ etc.). Despite of this, in the prescaling critical theory of HMF_3 no TGIMTs (1) are generated since the dynamical fermion mass vanishes in that case (for details, see ref. [11]).

Dynamical generation of TGIMTs was previously obtained for (2') [8]. However, no physical massive gauge bosons are produced in this case since conditions (9) are violated ($m_\psi = m_\varphi$, $\ell_{0,1}=\infty$). Similarly, the bound states generated by $\rho_{(0),A}$, σ_A disappear in (2'). The reason is that the (would-be) particle poles in the corresponding propagators (7.c,f),(8.a,d) coalesce with the two-particle ($\varphi-\varphi$, $\psi-\psi$ and $\psi-\varphi$) thresholds.

Finally, let us summarize the main results:

(a) $[\mathrm{GNLSM+F}]_3$ undergo second order phase transition connected with spontaneous breakdown of the internal $U(N)xU(n)_{gauge}$ symmetry (6).

(b) $[\mathrm{GNLSM+F}]_3$ exhibit dynamical P,T-breakdown in both "high-temperature" and "low-temperature" phases due to dynamical generation of m_ψ (6).

(c) In the "low-temperature" phase some fundamental fields (φ_{11} , ψ_{11}, $A^\mu_{(0)}$, $\underset{\sim}{A}^\mu$) are "confined" and standard Higgs mechanism does not operate.

(d) New gauge-invariant mass generation mechanism for $A^\mu_{(0)}$, $\underset{\sim}{A}^\mu$ is found: dynamical generation (7.c,f) of TGIMTs (1) without P,T-breaking in the classical theory (2).

(e) Massive composite fermions ("color" singlet $\rho_{(0)}$ and "color" vector ρ_A) and composite "color" vector bosons σ_A are explicitly obtained (8.a,d) as bound states of the classically massless fundamental fields, cf. (10), (γ).

The phenomena listed in (a)-(e) are of definite interest for the current phenomenology of realistic (D=4) gauge theories (for an extensive review, see [12]).

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Континуальный интеграл по траекториям в суперпространстве

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ЛЕНИНГРАДСКОЕ ОТДЕЛЕНИЕ МАТЕМАТИЧЕСКОГО ИНСТИТУТА АН СССР

Активное использование в современной теоретической физике систем, в которых равноправно фигурируют коммутирующие (бозевские) и антикоммутирующие (фермиевские) динамические переменные, ставит в ряд новых вопросов в рамках стандартных методов. Одним из наиболее известных и универсальных среди этих методов является континуальное интегрирование (см. например, /1,2/). Удобство этого метода исследования квантовых систем связано с тем, что в его формулировку входят в основном классические величины, допускающие наглядную физическую интерпретацию и позволяющие строить различные схемы теории возмущений на основе анализа классических уравнений движения. Эти положения претерпевают значительные изменения при расширении фазового пространства Γ коммутирующих динамических переменных (p,q) за счет добавления антикоммутирующих (грассмановых) координат (ξ).

Другая тенденция в теории элементарных частиц связана с рассмотрением систем, не имеющих выделенных координатных переменных и тем самым инвариантных относительно репараметризации (обще ковариантные системы). Действие для таких систем инвариантно при репараметризации траектории системы (мировой линии для скалярной частицы, мировой поверхности для струны и т.д.). Отличие обще ковариантных систем от систем с внутренней калибровочной симметрией приводит к модификации формализма собственного времени. Так действие массивной скалярной частицы с зарядом g, взаимодействующей с внешним электромагнитным полем $A_\mu(x)$ (реперный формализм)

$$S = -\frac{1}{2}\int_{t_1}^{t_2} d\tau \left(\frac{\dot{x}_\mu^2}{e} + em^2 + 2g\,\dot{x}_\mu A_\mu(x) \right) \tag{I}$$

инвариантно при замене $\tau \to \bar{\tau} = f(\tau)$, $f'(\tau) > 0$. При этом вспомогательная переменная (репер) также преобразуется $e \to \bar{e} = f'(\tau)e$.